ST HILDA'S
ANGLICAN SCHOOL FOR GIRLS INC.

Time: 16 minutes

Marks: 16

Total: 42

Mathematics Methods 3&4

Response Test 3 – Calculator Free

(Thursday August 19th)

ClassPad calculators are NOT permitted.

Formulae Sheet is permitted.

Name: ANSWERS

For Paperly Extra Time Forms:
ATMAM Response Test 3

Formula sheet is permitted for both parts.
Part 1 is resource free and so neither notes nor calculators are permitted.
Part 2 is resource rich and so half an A4 size page of notes and calculators are permitted.

1. [1 & 2 = 3 marks]

(a) Use base 10 logarithms to solve the equation $2^{3x} = 5$ exactly.

$\log(2^{3x}) = \log(5)$

$3x \log(2) = \log 5$ (1)

$x = \dfrac{\log(5)}{3\log(2)}$

3

(b) Solve the equation $5\log_2(3x-1) = 15$ giving your answer in simplest form.

$\log_2(3x-1) = 3$

$3x - 1 = 2^3$ (1)

$3x = 9$

$x = 3$ (1)

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2. [2 & 2 = 4 marks]

(a) Find $\dfrac{dy}{dx}$ in simplest form if $y = \ln(4\sin(3x))$.

$y = \ln 4 + \ln(\sin 3x)$ (1)

$\therefore \dfrac{dy}{dx} = 0 + \dfrac{3\cos 3x}{\sin 3x}$

$= \dfrac{3\cos 3x}{\sin 3x}$ (1)

or

$\dfrac{dy}{dx} = \dfrac{1}{4\sin(3x)} \cdot 12\cos(3x)$ (1)

$= \dfrac{3\cos 3x}{\sin 3x}$ (1)

4

(b) Find the exact value of k if $\int_1^7 \dfrac{2}{4x-3}\,dx = \ln(k)$

$\frac{1}{2}\int_1^7 \dfrac{4}{4x-3}\,dx = \ln k$

$\frac{1}{2}\left[\ln|4x-3| + c\right]_1^7 = \ln k$ (1) correct antidifferentiation

$\frac{1}{2}\left[\ln 25 - \ln 1\right] = \ln k$

$\frac{1}{2}\ln 25 = \ln k$

$\ln 25^{\frac{1}{2}} = \ln k$

$\ln 5 = \ln k$

$k = 5$ (1)

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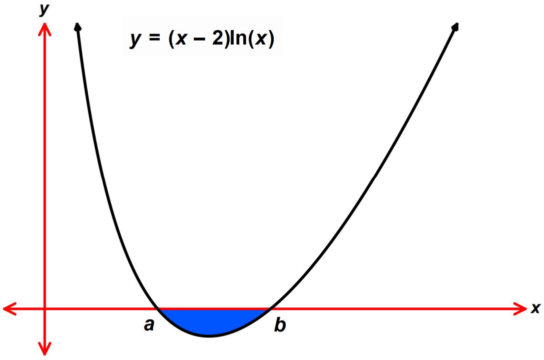
3. **[2, 4 & 3 = 9 marks]**

The curve with equation $y=(x-2)\ln(x),\ x>0$ is shown on the axes to the right.



(a) The graph has x-intercepts at $x=a$ and $x=b$. Determine the value of a and b.

$(x-2)\ln x = 0$ when

$x=2$ or $\ln x = 0$
$x=1$ (1) found both x-ints

So, $a=1$ and $b=2$ (1) $a<b$

(b) Find the equation of the tangent to the curve at the point where $x=b$.

$\frac{dy}{dx} = (1)(\ln(x)) + \left(\frac{1}{x}\right)(x-2)$ (0,1,2) correct differentiation (−1 each error)

When $x=2$, $y=0$ and $\frac{dy}{dx} = \ln 2 + \left(\frac{1}{2}\right)(0)$
$= \ln 2$ (1)

So, equation of tangent at $x=b$ is
$y-0 = \ln 2(x-2)$
$y=(\ln 2)x - 2\ln 2$ (1)

(c) The area of the shaded region between the curve and the x-axis is given by the definite integral $\int_c^d (x-2)\ln(x)dx$ which has the positive value of $\ln\left(4e^{\frac{5}{4}}\right)$.

$\frac{}{9}$

(i) State the value of c.

Area $= \int_b^a (x-2)\ln(x)\,dx$
$\therefore c=2$ (1)

(ii) The area of the shaded region $\ln\left(4e^{\frac{5}{4}}\right)$ can be expressed in the form $p\ln(q)+r$. Find the exact value of the rational constants p, q and r.

Area $= \ln\left(4e^{\frac{5}{4}}\right)$
$= \ln 4 + \ln\left(e^{5/4}\right)$
$= \ln 2^2 + \frac{5}{4}\ln e$ (1)
$= 2\ln 2 + \frac{5}{4}$

So, $p=q=2$ and $r=\frac{5}{4}$ (p, q and r are rational)
(1)

End of Resource Free

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ST HILDA'S
ANGLICAN SCHOOL FOR GIRLS INC.

Time: 28 minutes

Marks: 26 marks

Mathematics Methods 3&4

Response Test 3 – Calculator Assumed

(Thursday August 19th)

Half an A4 page of notes and ClassPad calculators are permitted.

Formulae Sheet is permitted.

Name: ANSWERS

4. **[1, 2, 2 & 1 = 6 marks]**

The number n of patients with a disease t weeks after commencing a course of treatment is modelled by $n(t) = 50 + 50\ln(e - t)$, $0 \le t \le b$.

(a) How many patients have the disease initially?

$n(0) = 50 + 50\ln(e - 0)$
$= 100$

(1)

So, 100 patients initially have the disease

(b) To the nearest day, how many days after commencing treatment are there 20 patients with the disease?

$n(t) = 20$ when $50 + 50\ln(e - t) = 20$
$t = 2{\cdot}169\ldots$ weeks (1)

So, after 15 days (nearest day)
(1)

6

(c) Correct to the nearest whole number, what is the rate of change of n when $t = 1{\cdot}5$

$\frac{dn}{dt} = -\frac{50}{e - t}$ (1)

When $t = 1.5$, $\frac{dn}{dt} = -41.041\ldots$

So,
when $t = 1.5$, number of patients with the disease is decreasing at the rate of 41 people/week
(1) rounding (nearest whole)

(d) The model ceases to be valid when all patients are cured. Determine the exact value of b.

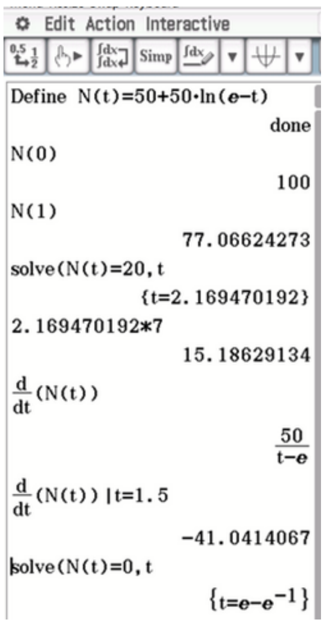
All cured when $n(t) = 0$

$50 + 50\ln(e - t) = 0$

$t = e - e^{-1}$ weeks
$(\approx 2.350\ldots)$

So, $b = e - \frac{1}{e}$ (1) exact required

or $\frac{e^2 - 1}{e}$



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5. **[1, 2 & 5 = 8 marks]**

A company has ten telephone lines. At any instant, the probability that any particular line is engaged (in use) is $\frac{1}{5}$. Let X = the number of the ten telephone lines that are free.

(a) State the type of probability distribution that X follows including the values of relevant parameters.

$X \sim Bi\left(n = 10,\ p = P(\text{free}) = \frac{4}{5}\right)$ (1) must include value of n and p

(b) **(i)** State the expected number of free (not in use) telephone lines.

$\text{Expected} = 10 \times \frac{4}{5}$

$= 8$ lines free (1)

(ii) Find the variance of the number of free telephone lines.

$\text{Variance} = 10 \times \frac{4}{5} \times \frac{1}{5}$

$= \frac{8}{5}$ or 1.6 (1)

(c) Calculate, correct to 3 decimal places, the probability that

(i) 4 of the lines are engaged

$= P(X = 6)$

$= 0.08808\ldots$ (1)

$= 0.088$ (3dp's)

(ii) at least 4 lines are free

$= P(X \geq 4)$

$= 0.99913\ldots$ (1)

$= 0.999$ (3dp's)

(iii) at least 6 lines are free if at least 4 lines are free

(1) answers rounded to 3dp's in at least 2 of Q5(c)

$= P(X \geq 6 \mid X \geq 4)$

$= \frac{P(X \geq 6 \cap X \geq 4)}{P(X \geq 4)}$

$= \frac{P(X \geq 6)}{P(X \geq 4)}$

$= \frac{0.9672065\ldots}{0.9991356\ldots}$ (1) for $P(X \geq 6)$

$= 0.96804\ldots$ (1)

$= 0.968$ (3dp's)

$\frac{\ }{8}$

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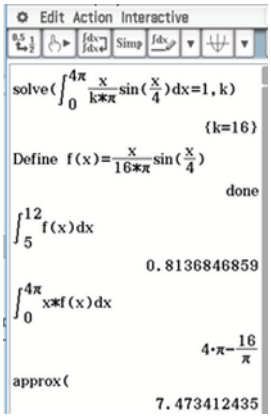
6. **[3, 1 & 2 = 6 marks]**

The temperature, X degrees Celsius inside a refrigerator has been found to have a probability density function $f(x) = \begin{cases} \dfrac{x}{k\pi}\sin\left(\dfrac{x}{4}\right) & , \ 0 \le x \le 4\pi \\ 0 & , \ \text{elsewhere} \end{cases}$ where k is a constant.



(a) Find

(i) the value of k

$$\int_0^{4\pi} \frac{x}{k\pi}\sin\left(\frac{x}{4}\right)dx = 1 \quad (1)$$

$$k = 16 \quad (1)$$

(ii) the probability that the refrigerator's temperature is between $5^\circ C$ and $12^\circ C$

$$P(5 < X < 12) = \int_5^{12} \frac{x}{16\pi}\sin\left(\frac{x}{4}\right)dx$$

$$= 0.8137 \ (4 \text{ dp's}) \quad (1)$$

(b) Calculate the exact mean temperature inside this refrigerator.

$$\text{Mean } (\bar{x}) = \int_0^{4\pi} x.f(x)\,dx$$

$$= 4\pi - \frac{16}{\pi} \quad (1) \text{ exact value required}$$

$$\approx 7.5^\circ C \ (1 \text{ dp})$$

$\overline{}$
6

(c) Calculate the standard deviation of the temperature inside this refrigerator correct to three decimal places.

$$\text{Variance} = \int_0^{4\pi} (x - \bar{x})^2 . f(x)\,dx$$

$$= 6.0617769... \quad (1)$$

So, $\text{Standard Deviation} = \sqrt{\text{Variance}}$

$$= 2.462^\circ C \ (3 \text{ dp's})$$

(1) don't penalise not rounding to 3dp's.

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7. [3, 1 & 2 = 6 marks]

The intelligence quotient or IQ, as measured by IQ tests, is a normally distributed random variable with mean of 100 and standard deviation of 15.

There are currently 10000 members of the West Coast Eagles.

(a) How many of the 10000 members of the West Coast Eagles would be expected to have an IQ that is

(i) between 90 and 120?

$= 10000\, P(90 \leq IQ \leq 120)$ (1)
$= 10\,000\,(0{\cdot}656296\ldots)$
$= 6562{\cdot}96\ldots$ So, 6563 members

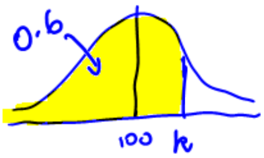
(ii) over 130?

$= 10000\, P(IQ \geq 130)$ (1)
$= 10000\,(0{\cdot}02275013\ldots)$
$= 227{\cdot}5013$

So, 228 members

(1) rounded correctly to nearest whole number

(b) Find the 0·6 quantile of IQ's of the members of the West Coast Eagles.



$P(IQ < k) = 0{\cdot}6$
$k = 103{\cdot}8\ldots$ (1)
So, 0·6 quantile is 103·8

$\frac{}{6}$

(c) If four of the 10000 members of the West Coast Eagles are randomly selected, what is the probability that exactly one of the four has an IQ over 130?

Let W = the number of West Coast Eagles members from a group of 4 members that have IQ > 130

then $W \sim Bi\,(n = 4,\ p = P(IQ > 130)$ $)$ (1)
$= 0{\cdot}02275013\ldots$

$P(\text{one of the four has } IQ > 130) = P(W=1)$
$= 0{\cdot}0849$ (4 dp's)
(1)

End of Resource Rich

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